Universal Gravitation:!!

The local Acceleration due to gravity depends on the Mass of the planet you are standing on, as well as its radius.

They are related the following way: $g = \frac{GM}{r^2}$

Where $G = 6.67 \times 10^{-11}$

On Earth $g = \frac{GM_e}{r_e^2} = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.38 \times 10^6)^2} \approx 9.8 \ m/s^2$

The Force of Gravity is Calculated by $F_g = mg$

So, in general $F_g = m\left(\frac{GM}{r^2}\right) = \frac{GMm}{r^2}$

This is the Law of Universal Gravitation and it works for any two masses, regardless of how far apart they are.

Examples:

a) Calculate the force of gravity acting on a 100 kg astronaut, 100 km above the Earth's Surface.

$F_g = \frac{GMm}{r^2} = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(100)}{(6.38 \times 10^6 + 100000)^2}$

$F_g = \frac{3.98866 \times 10^{16}}{(6.48 \times 10^6)^2}$

$F_g = 949.90 N.$

b) Compare the force on the astronauts 100 km up to the force they would feel on the Earth's Surface

On Earth $\rightarrow F_g = 9.8(100) = 980 N$

As a Percent: $\frac{949.9}{980} = 96.9\%$

You still experience 96.9% of Earth's Gravity 100 km up.

c) CALCULATE THE FORCE OF GRAVITY between two oil tankers ($m = 6 \times 10^8$ kg) that are 50 m apart.

$$F_g = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11})(6\times10^8)(6\times10^8)}{(50)^2}$$

$$F_g = \frac{24012000}{2500}$$

$$F_g = 9604.8N.$$

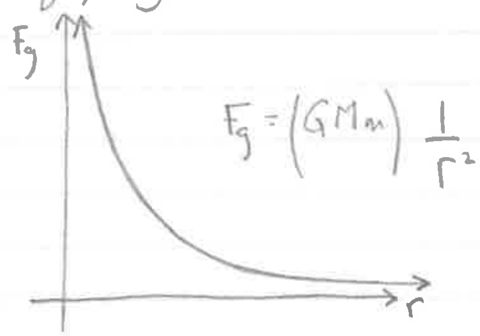
d) CALCULATE THE FORCE OF GRAVITY A 100kg ASTRONAUT WOULD FEEL ON THE MOON + COMPARE IT WITH EARTH'S

$$F_g = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11})(7.35\times10^{22})(100)}{(1.74\times10^6)^2}$$

$$F_g = \frac{4.90245\times10^{14}}{3.0276\times10^{12}} = 161.93N$$

$$\frac{161.93}{980} = 16.5\%$$

If you graph the Force vs r:



If you sketch the $\frac{1}{r^2}$ function you see that the force of gravity decreases quite rapidly as you get farther apart.

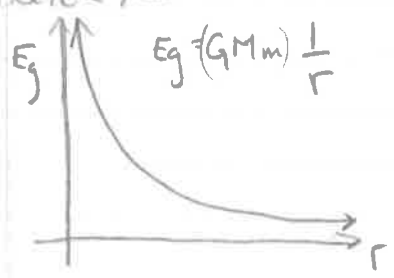
The Potential Energy near earth is Calculated by $E_g = mgh$.

So, in General $E_g = m\left(\frac{GM}{r^2}\right)h$ ← but height is really a measure of radius...

let $h = r$

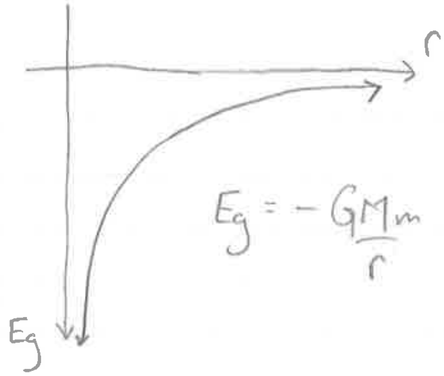
So $E_g = \frac{GMmr}{r^2} = \frac{GMm}{r}$.

Sketch this:



This shows the potential energy decreasing as you get farther away (which is backwards)

Add a negative to the equation to reflect it across the x-axis.



This graph shows the amount of energy increasing as you go farther away (which we want)

But it makes the energy negative.

This is interpreted as an energy debt, or energy well.

It won't matter too much, because every calculation we will do is a difference in energy from one place to another

In General: $\Delta E_g = E_{g_{END}} - E_{g_{START}}$

$$\Delta E_g = -\frac{GMm}{r_{END}} - \left(-\frac{GMm}{r_{START}}\right)$$ Factor this!

$$\Delta E_g = -GMm\left(\frac{1}{r_{end}} - \frac{1}{r_{start}}\right)$$ Find a Common Denominator

$$\Delta E_g = -GMm\left(\frac{r_{START}}{r_{START}\, r_{end}} - \frac{r_{end}}{r_{end}\, r_{START}}\right)$$

$$\therefore \quad \Delta E_g = GMm\left(\frac{r_{end} - r_{start}}{r_{end}\, r_{start}}\right)$$

Example: Calculate the energy required to lift a 100 kg astronaut 100 km above the surface of EARTH (where the ISS is).

$r_{START} = r_e = 6.38 \times 10^6$

$$\Delta E_g = \frac{(6.67\times10^{-11})(5.98\times10^{24})(100)(6.48\times10^6 - 6.38\times10^6)}{(6.48\times10^6)(6.38\times10^6)}$$

$r_{end} = r_e + 100000$
$= 6.48 \times 10^6$

$\Delta E_g = 96478675.6\,J \approx 96.5\,MJ$ (MegaJoules)
$\approx 965\,kJ$ per kilogram.

Escape Velocity!

In the Special Case where $r_{end} = \infty$ the ΔE_g equation is a bit different

$$\Delta E_g = -GMm\left(\frac{1}{r_{end}} - \frac{1}{r_{START}}\right)$$

$$\Delta E_g = -GMm\left(\frac{1}{\infty}^{\nearrow 0} - \frac{1}{r_{START}}\right) = -GMm\left(\frac{-1}{r_{START}}\right)$$

$$\therefore E_g = \frac{GMm}{r}$$

This is the "Escape Energy" since, once you are infinitely far away from an object, it's unlikely that you are coming back!

Example: Calculate the amount of energy a 100kg astronaut needs to "escape" the Earth, and compare it to the amount of energy needed to reach low orbit (100km up; see previous example)

Escape Energy: $E_g = \frac{GMm}{r} = \frac{(6.67\times10^{-11})(5.98\times10^{24})(100)}{6.38\times10^{6}}$ ← you are escaping from EARTH's Surface.

$E_g = 6.25\times10^{9}\,J$

$E_g = 6.25\,GJ$ (gigaJoules ü)

$E_g = 62.5\,MJ$ per kilogram.

It takes 965kJ (=0.965MJ) to reach low orbit...

$\therefore$ It takes $\frac{62.5}{0.965} \approx 65$ times more energy to "escape" Earth.

If we assume that all of this energy comes from Kinetic Energy,

$$E_K = \Delta E_g \quad \text{(escape energy)}$$

$$\tfrac{1}{2}\cancel{m}v^2 = \frac{GM\cancel{m}}{r}$$

$$v^2 = \frac{2GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}} \leftarrow \text{The escape velocity}$$

any object that achieves escape velocity for a planet or moon will zip off into space, never to return.

Example: Calculate the escape velocity for Earth + the moon

For Earth:

$$v = \sqrt{\frac{2(6.67\times10^{-11})(5.98\times10^{24})}{6.38\times10^{6}}}$$

$$v = \sqrt{125036363.63\ldots}$$

$$v \approx 11181.966 \tfrac{m}{s}$$

$$v \approx 11.18 \tfrac{km}{s}$$

For the moon:

$$v = \sqrt{\frac{2(6.67\times10^{-11})(7.35\times10^{22})}{1.74\times10^{6}}}$$

$$v = \sqrt{5635000}$$

$$v \approx 2373.815 \tfrac{m}{s}$$

$$v \approx 2.37 \tfrac{km}{s}$$

kilometers per second :)

Example: How small would you need to compact the earth so that even light could not escape?

$v = 3\times10^{8}\ \tfrac{m}{s}$ (speed of light)

$$v^2 = \frac{2GM}{r}$$

$$\therefore r = \frac{2GM}{v^2}$$

$$r = \frac{2(6.67\times10^{-11})(5.98\times10^{24})}{(3\times10^{8})^2}$$

$r = 0.008863\,m$

$r = 8.9\,mm$

$\therefore$ diameter = 17.7 mm

If you could squash the entire Earth into a sphere 17.7mm across, even light wouldn't be able to escape! (This is a Black Hole)

CALCULATE THE FORCE OF GRAVITY BETWEEN EDWINA (63 kg) AND ARCHIBALD (86 kg) WHEN THEIR CENTRES OF MASS ARE 4 m APART.

CALCULATE THE ACCELERATION DUE TO GRAVITY ON THE SURFACE OF MARS. ($m_{MARS} = 6.48 \times 10^{23}$ kg, $r_{MARS} = 3.39 \times 10^{6}$) COMPARE YOUR ANSWER WITH EARTH'S.

CALCULATE THE AMOUNT OF ENERGY per kilogram THAT IS REQUIRED TO LIFT OBJECTS from the SURFACE of MARS TO LOW ORBIT (100 km above the surface)

Calculate THE ESCAPE ENERGY for the PLANET MARS. COMPARE YOUR ANSWER TO THE "GET TO ORBIT" ENERGY.

CALCULATE THE ESCAPE Velocity for MARS, AND COMPARE IT TO THAT of the EARTH + the MOON.

TAKE ALL of the things ON MARS, AND COMPRESS THEM INTO A SMALLER SPHERE SO THAT THE SURFACE GRAVITY INCREASES. CALCULATE THE NEW MARS radius that would make $g_{MARS} = g_{EARTH}$.